

# Comprehensive Optimum Seeking Method Based on the Set Pair Analysis and Hierarchy for Architectural Design

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**Abstract:** Set pair analysis can solve uncertainty problem of system. In this paper, it adopts set pair analysis principle, hierarchical analysis and set-valued statistical method to establish the evaluation model by considering the weight of influencing factors, so as to discuss the evaluation method of architectural design scheme. Moreover, it is verified by examples.

## 1. Introduction

The completion of housing architectural design scheme involves many specialties. Among the multiple design schemes of the same building, some put focus on appearance, some put focus on use function, while others have good building economic indicators. At present, expert evaluation method is widely used in the comprehensive evaluation of architectural design schemes, which can better analyze and compare different design schemes of the same building, but due to the characteristics of multi-disciplinary, it will inevitably be affected by more human factors. How to evaluate the advantages and disadvantages of an architectural design scientifically and accurately is a multi-objective decision-making problem.

Based on the principle of set pair analysis, this paper synthetically evaluates the architectural design schemes by multiple factors. On the basis of determining the weight distribution among the influencing factors, it explores the comprehensive evaluation model as well as method of architectural design schemes, so as to reduce the influence of human factors in the evaluation of architectural schemes.

## 2. Optimization Principle of Set Pair Level

The evaluation of architectural design must solve the problem of single index evaluation and multi index comprehensive evaluation. When evaluating a single index, the qualitative indicators in the evaluation system are generally determined by experts' scoring. In order to solve the problem that experts can not easily give an absolute single value during evaluating, the set-valued statistical method can be used to determine the evaluation value of single index, so that the scoring made by expert can give a scoring interval.

In the multi index comprehensive evaluation, we must firstly determine the weight of each index to the target. Analytic hierarchy process (AHP) is an effective method to determine the weight of complex system. It can transform complex systems into several simple systems by hierarchical decomposition. By calculating the weight of each index in a simple system relative to the upper criteria, the single ranking weight of each index can be determined. The problem of determining the weight of the lowest index to the highest index, namely, the weight of target can be solved by using the composite weight calculation method.

Set pair analysis theory divides certainty into two aspects: "identity" and "opposition". Uncertainty is called "difference". It analyses things and their systems from three aspects: similarity, difference and opposition. It is believed that similarities, differences and opposites are interdependent, interrelated and mutually restrictive, which can be transformed under certain conditions. In the context of a certain problem, the relation between the same set, the different set

and the opposite set can be expressed by the connection degree  $\mu$ .

In the formula,  $N$  is the total number of set-pair characteristics;  $S$  is the same number of set-pair characteristics;  $P$  is the opposite number of set-pair characteristics;  $F$  is the number of set-pair characteristics that are neither different nor opposite,  $F = N - S - P$ ;  $\frac{S}{N}$ ,  $\frac{F}{N}$ ,  $\frac{P}{N}$  are the same degree, difference degree and opposition correspondingly;  $i$  is the number of difference degree indicators,  $i$  is valued in the interval of  $[-1, 1]$ ;  $j$  is the number of opposition degree indicators, in general, taken as  $j = -1$ . The identical degree of set pair is the depiction of the convergence degree of two sets under the specified background.

$$\mu = \frac{S}{N} + \frac{F}{N}i + \frac{P}{N}j \quad (1)$$

The basic idea of architectural design scheme set is to use is to determine the weight of each evaluation index by analytic hierarchy process (AHP), so as to determine the evaluation value of single index by set-valued statistics method, forming a set pair of evaluation schemes and ideal schemes by set pair analysis theory, and selecting the optimal scheme through the same degree analysis of both schemes.

### 3. Evaluation Model of Architectural Design Scheme

#### 3.1 Evaluation index system of architectural design scheme

Architectural design schemes can be influenced by many factors, such as building function, building technology, building economy, building equipment, building image and environmental protection, etc. The basic criterion ( $y_r$ ) can constitute the criterion layer of the evaluation index system of architectural design schemes, and each criterion restricts and influences each other. Each criterion also contains many factors, including both quantitative indicators, such as project cost, and qualitative indicators, such as the rationality of design schemes, the impact of buildings on the environment, etc. These indicators can constitute the index layer ( $x_i$ ) of the evaluation index system of architectural design schemes.

A comprehensive evaluation index system for building design can be established, shown in Fig.1.

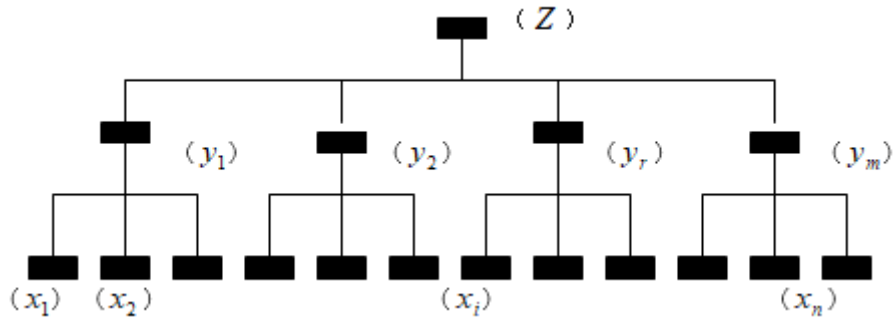


Fig. 1 Evaluation Index System of Architectural Design Scheme

#### 3.2 Determination of single index evaluation value

There are quantitative indicators and qualitative indicators in the evaluation system of architectural design scheme. Quantitative indicators can be evaluated by their actual values, such as engineering cost, energy consumption, etc. For qualitative indicators, experts can be employed to score by percentage system. In order to avoid the difficulty caused by a single score, experts can give an evaluation interval value when scoring, and then determine the evaluation value by using set-valued statistical method.

If there is  $L$  experts participating in the evaluation, evaluation on No.  $k$  scheme ( $k = 1, 2, \dots, K$ ,  $K$  is the number of the scheme), as well as the interval  $[t_{ij1}^{(k)}, t_{ij2}^{(k)}]$  is given by No.  $j$  expert ( $j = 1, 2, \dots, L$ ) to No.  $i$  index ( $i = 1, 2, \dots, n$ ,  $n$  is the number of index), then the set-valued statistical evaluation value  $t_i^{(k)}$  of the first No.  $i$  index of  $k$  scheme is

$$t_i^{(k)} = \frac{\frac{1}{2} \sum_{j=1}^m [(t_{ij2}^{(k)})^2 - (t_{ij1}^{(k)})^2]}{\sum_{j=1}^m [t_{ij2}^{(k)} - t_{ij1}^{(k)}]} \quad (i = 1, 2, \dots, n; \quad k = 1, 2, \dots, K) \quad (2)$$

### 3.3 Determining weight of evaluation index by AHP

According to the hierarchical structure model shown in Fig.1, the scaling method of 1-9 and its reciprocal can be adopted at each level according to certain criteria. Several experts are employed to compare the importance of each index in pairs, and a judgment matrix can be constructed. The maximum eigenvalue  $\lambda_{\max}$  and eigenvector  $\omega$  of the judgment matrix can be calculated, and the weight of the layer can be determined after consistency test. If the single ranking weight of criterion  $r$  to target level  $Z$  is  $\omega Y_r$  ( $r = 1 \sim m$ ), and the single ranking weight of index  $i$  to criterion level  $r$  is  $\omega X_{ir}$ , then the weight of index  $X_i$  to target level  $Z$  is  $\omega_i$ .

$$\omega_i = \sum_{r=1}^m (\omega Y_r \cdot \omega X_{ir}) \quad (i = 1, 2, \dots, n) \quad (3)$$

### 3.4 Optimize architectural design scheme by using set pair analysis principle.

Among the indicators of evaluation system of architectural design schemes, some are benefit-oriented indicators, namely, the bigger, the better, while others are cost-oriented indicators, namely, the smaller, the better.

An ideal scheme  $A_0$  can be determined by  $K$  architectural design schemes to be evaluated, and a set pair of the schemes to be evaluated and the ideal scheme can be formed. The value of each index in ideal scheme  $A_0$  should be the optimum value in the evaluated  $K$  schemes, that is, the maximum value in the same kind of index for the benefit type index and the minimum value in the same kind of index for the cost type index. If the evaluation value  $t_i^{(k)}$  of No.  $i$  index in No.  $k$  scheme,  $a_i^{(k)}$  is the same degree of the optimal value  $t_0$  with the corresponding index in the ideal scheme  $A_0$ , it can be obtained from the concept of identity degree in set pair analysis theory.

The benefit- oriented indicators ( $t_i^{(k)} < t_0$ ):

$$a_i^{(k)} = \frac{t_i^{(k)}}{t_0} \quad (4)$$

The cost- oriented indicators ( $t_i^{(k)} > t_0$ ):

$$a_i^{(k)} = \frac{t_0}{t_i^{(k)}} \quad (5)$$

After combining the weight of each index  $\omega_i$ , the same degree between the evaluation scheme and the ideal scheme  $a^{(k)}$  is:

$$a^{(k)} = \sum_{i=1}^n \omega_i a_i^{(k)} \quad (k = 1, 2, \dots, K) \quad (6)$$

The order of advantages and disadvantages of  $K$  schemes can be determined by the size

of  $a^{(k)}$  value. The larger  $a^{(k)}$  value is, the better the scheme is.

#### 4. Example analysis

A building complex has three architectural design schemes: A, B and C, which needs to be compared comprehensively. Five engineering experts from the related fields are invited to grade these three schemes according to their requirements, and then they carry out a comprehensive evaluation.

##### 4.1 Influencing factors

According to the characteristics of the schemes to be evaluated, the criterion layer factors can affect the three schemes, which can be determined as functional index ( $y_1$ ), technical index ( $y_2$ ) and economic index ( $y_3$ ). Among them, the function index should consider space combination ( $x_1$ ), lighting and ventilation ( $x_2$ ); technical index should consider structure form ( $x_3$ ), construction scheme ( $x_4$ ); economic index should consider project cost ( $x_5$ ), land use ( $x_6$ ), energy consumption ( $x_7$ ).

##### 4.2 Determining the single index evaluation value and its ideal value of each evaluation scheme

In the evaluation index system, both  $x_5$  and  $x_6$  are quantitative indicators, taking their actual value as the evaluation value.  $x_5$  can be taken as the value of the unit cost (10,000 yuan), while  $x_6$  can be taken as the value of volume ratio. For the other qualitative indicators, the evaluation experts can give the scoring interval according to the percentage system, by using the set-valued statistical method to calculate the evaluation value according to Formula (2). The ideal value of each index in ideal scheme  $A_0$  is the optimal value of the corresponding evaluation index in the three schemes. The evaluation value of each single index and its ideal value can be detailed in Table 1.

Table 1 Single index evaluation value and its ideal value

Basic criteria	Index factor	Index type	Single index evaluation value			Ideal value
			$A_1$	$A_2$	$A_3$	
Functional indicators( $y_1$ )	Spatial combination( $x_1$ )	Benefit type	85.62	91.14	81.37	91.14
	Lighting ventilation( $x_2$ )	Benefit type	83.43	85.37	89.22	89.22
Technical indicators( $y_2$ )	Structural form( $x_3$ )	Benefit type	87.63	92.56	89.91	92.56
	Construction scheme( $x_4$ )	Benefit type	91.23	88.74	86.52	91.23
Economic indicators( $y_3$ )	Project cost( $x_5$ )	Cost type	0.30	0.31	0.29	0.29
	Land use( $x_6$ )	Benefit type	4.31	4.45	4.41	4.45
	Energy consumption( $x_7$ )	Benefit type	90.67	91.84	92.17	92.17

##### 4.3 Using AHP to determine the weight of each index.

The weight of index layer to criterion layer and criterion layer to target layer can be calculated by using analytic hierarchy process. The weight  $\omega_i$  of index layer to target layer can be calculated by Formula (3). The result can be shown in Table 2.

##### 4.4 Using set pair analysis theory to calculate the same degree of the evaluation scheme and the ideal scheme

According to the evaluation value and ideal value of each index in Table 1, by Formula (4) and

Formula(5), it can calculate the same degree  $a_i^{(k)}$  ( $k = 1, 2, 3$ ) of each evaluation value index and its corresponding ideal value in these three schemes respectively. The result can be detailed in Table 2.

Table 2 The same degree  $a$  of each evaluation index and its ideal value, as well as index weight  $b$ .

$k$	$a_i^{(k)}$	$x_i$						
		1	2	3	4	5	6	7
1	$a_i^{(1)}$	0.939	0.935	0.947	1	0.967	0.969	0.984
2	$a_i^{(2)}$	1	0.957	1	0.973	0.935	1	0.996
3	$a_i^{(3)}$	0.893	1	0.971	0.948	1	0.991	1
Weight	$\omega_i$	0.201	0.092	0.138	0.129	0.12	0.145	0.175

Considering the weight  $\omega_i$  of each evaluation index, Formula (6) can calculate the identity degree  $a^{(k)}$  of each evaluation scheme and the ideal scheme, ranking the advantages and disadvantages of each scheme. The results can be shown in Table 3.

Table 3 Same Degree of Each Index and Ideal Scheme and Ranking of Advantages and Disadvantages

$k$	Scheme 1	Scheme 2	Scheme 3
$a^{(k)}$	0.963	0.984	0.966
Ranking	2	3	1

According to the calculation result, Scheme 3 is the best, followed by Scheme1 and Scheme 2

## 5. Conclusion

This paper is based on set pair analysis theory, combined with set-valued statistics and hierarchical and analytic method, so as to establish a new comprehensive evaluation and optimization model for architectural design schemes. The application of set-valued statistics can provide a more flexible space for evaluation experts to grade qualitative indicators, which can improve the scientificity of expert judgment on problems. The application of hierarchical analysis and composite weight calculation method can better solve the impact of the lowest level indicators on evaluation objectives in complex multi-attribute hierarchical structure and improve the synthesis, so as to improve the accuracy of evaluation. The calculation examples show us that the method can have clear thinking, with simple calculation, which can have practical application value.

## References

- [1] Zhao Keqin. Set Pair Analysis and Its Preliminary Application. Hangzhou. Zhejiang Science and Technology Press, 2000.
- [2] Wang Jing, Su Wei. Comprehensive Optimization of Deep Foundation Pit Support Schemes Based on Set Pair Hierarchy Theory. Mathematical Practice and Understanding. No. 3, 2006. P142-146.
- [3] Su Wei, Wang Jing. Fuzzy Hierarchical Evaluation Model and Method for Architectural Design Schemes. Henan Science. No. 2, 2002. P161-164.